

# Surface Water Quality Modeling

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<https://triton.wasser.tum.de/>



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# *Water quality under climate change* TUM



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# Content

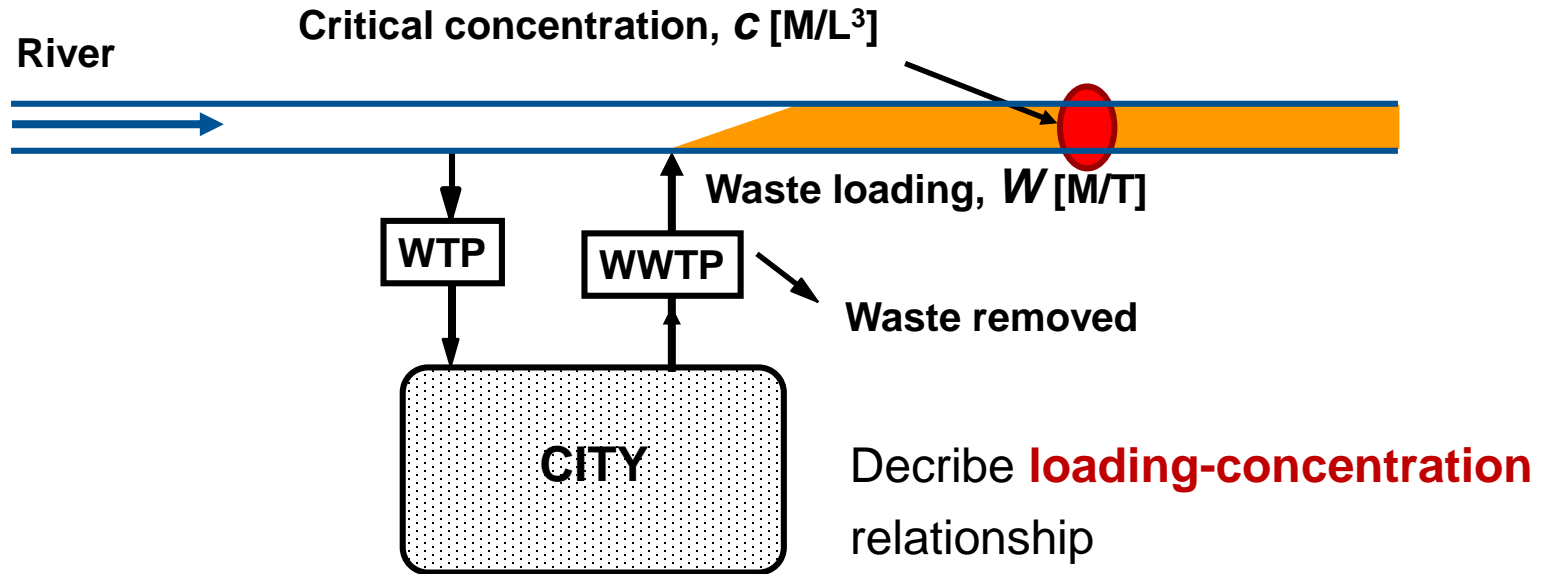
1. Introduction to Water Quality Modeling
  - What is water quality modeling?
  - Mass balance
  - Analytical & numerical solutions
2. Kinetics
  - Zero & first-order reactions
  - Temperature effects
3. Applications



# ***P1. Introduction to Water Quality Modeling***

- ☀ ***What is water quality modeling?***
- ☀ ***Mass balance***
- ☀ ***Analytical & numerical solutions***

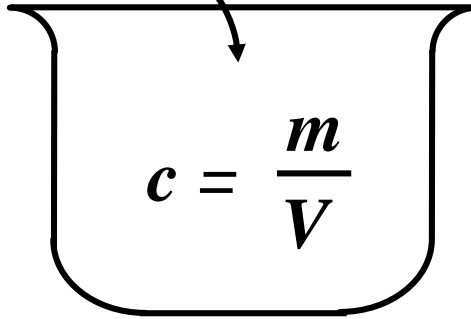
# URBAN POINT SOURCE DESIGN PROBLEM (Circa 1920s)



**WATER-QUALITY  
MODEL:**  $C = f(W, \text{physics, chemistry, biology})$

# MASS AND CONCENTRATION

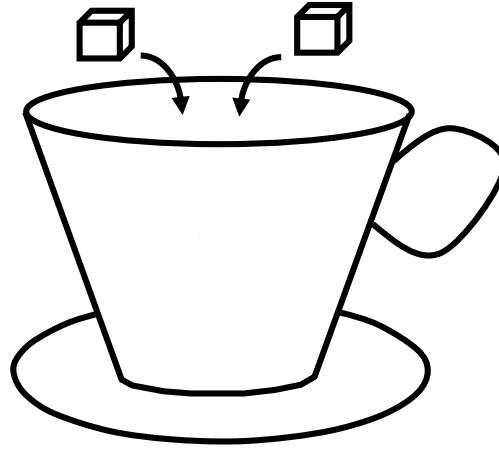
$m$  = mass



$V$  = volume

units:  $\frac{\text{M}}{\text{L}^3}$

*note: M = mass, L = length, T = time, H = heat*

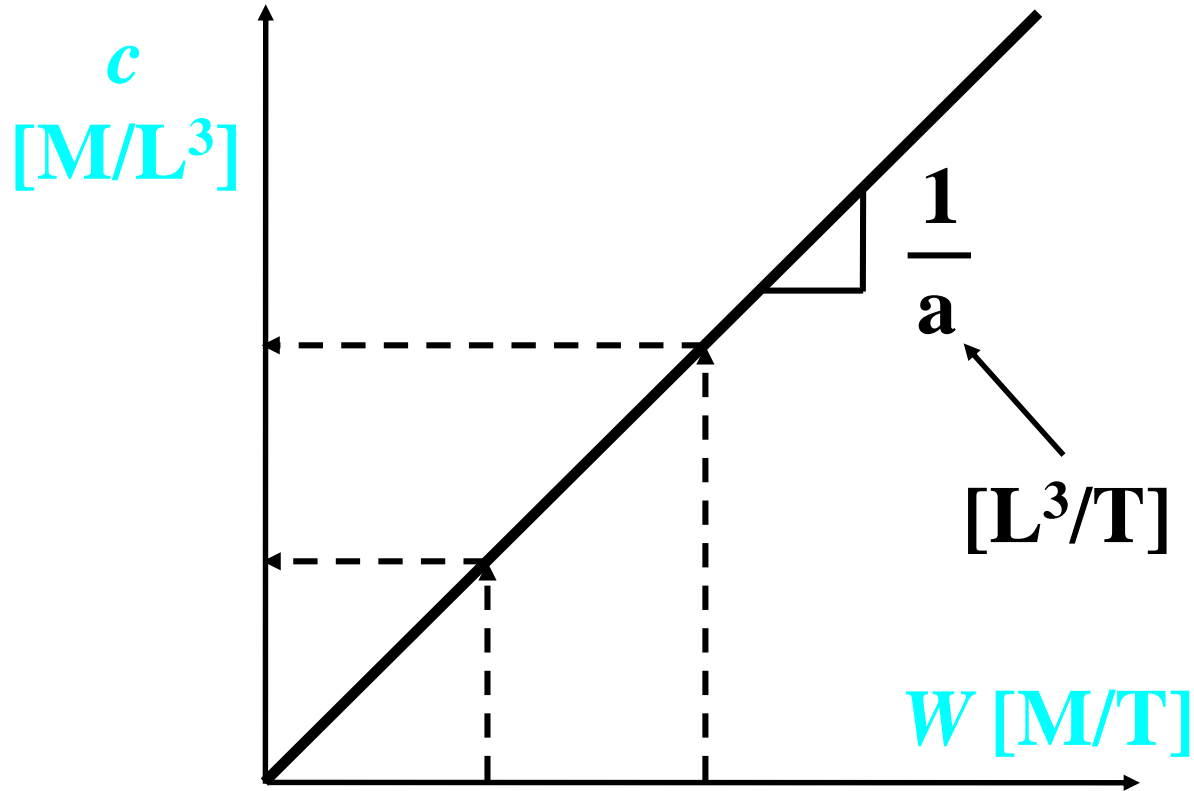


**MASS → AMOUNT OF SUGAR**

**CONCENTRATION → SWEETNESS**

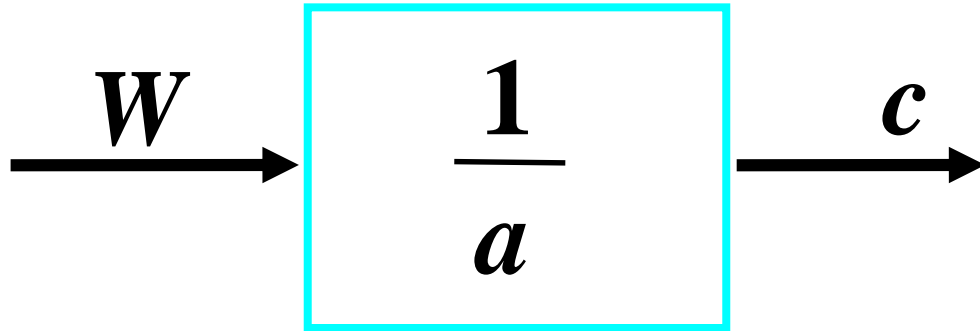


# ***SIMPLE LINEAR MODEL*** TUM



# ASSIMILATION FACTOR

$$c = \frac{1}{a} W$$



stimulus  $\rightarrow$  system  $\rightarrow$  response



physics, chemistry, biology

# MODELING MODES

***SIMULATION MODE: Given load ( $W$ ) and assimilation factor ( $a$ ), calculate***

$$c = \frac{1}{a} W$$

***ASSIMILATIVE CAPACITY DESIGN MODE: Given desired concentration ( $c$ ) and assimilation factor ( $a$ ), calculate***

$$W = a c$$

***ENVIRONMENTAL MODIFICATION DESIGN MODE: Given desired concentration ( $c$ ) and load ( $W$ ), calculate***

$$a = \frac{W}{c}$$

# HOW DO WE DETERMINE

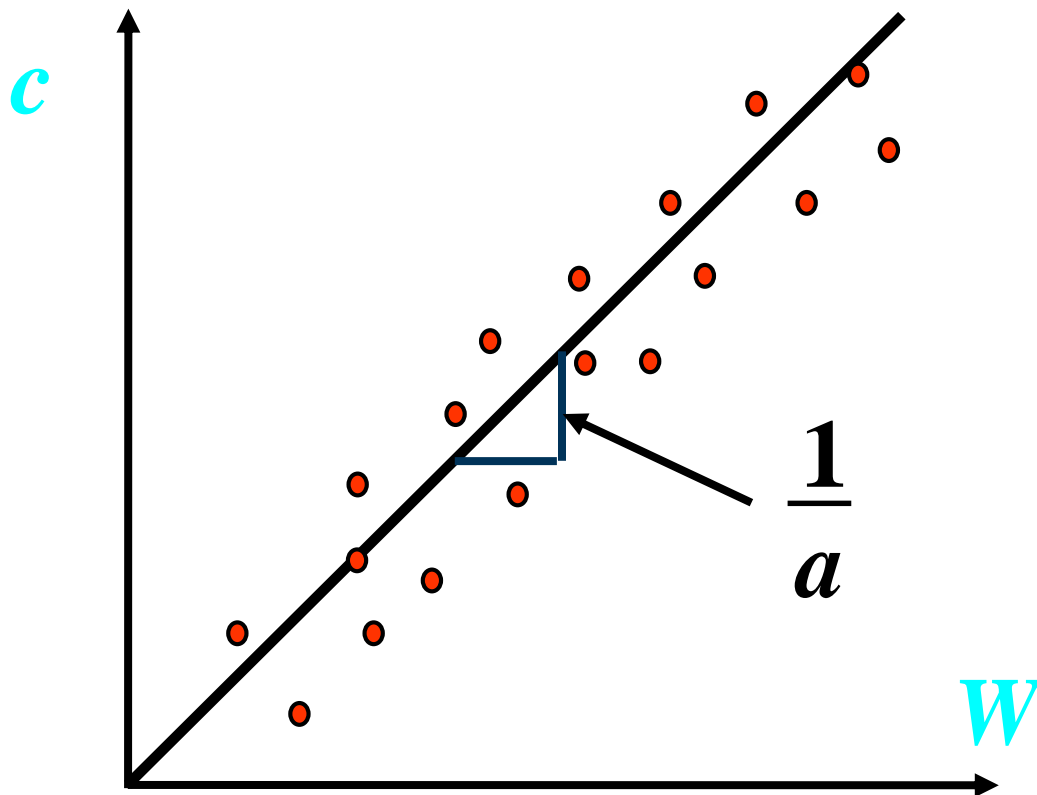
*“a”?*



***EMPIRICAL***  
***Data-based***  
***(Inductive,***  
***Statistical)***  
***Model***

***MECHANISTIC***  
***Process-based***  
***(Deductive***  
***Mass-balance)***  
***Model***

# EMPIRICAL MODELS



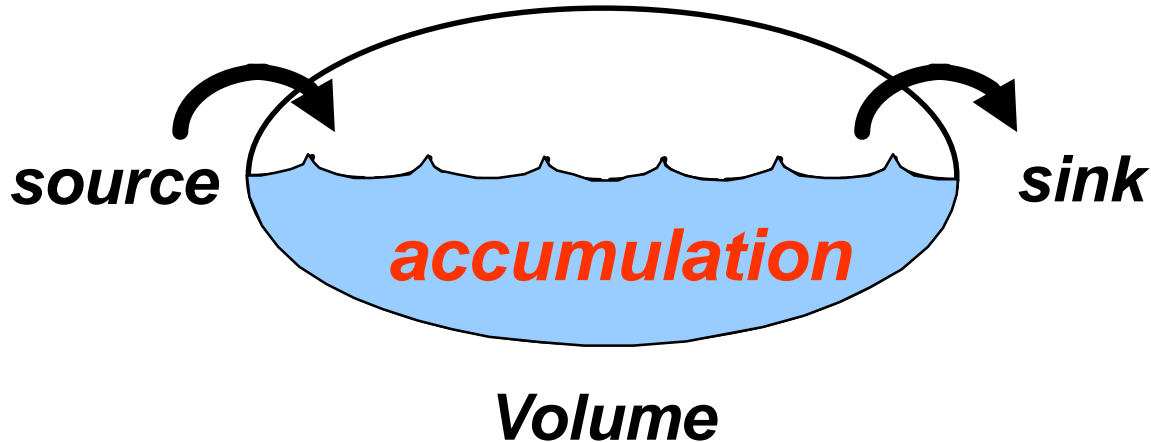
# MECHANISTIC MODELS

## CONSERVATION OF MASS

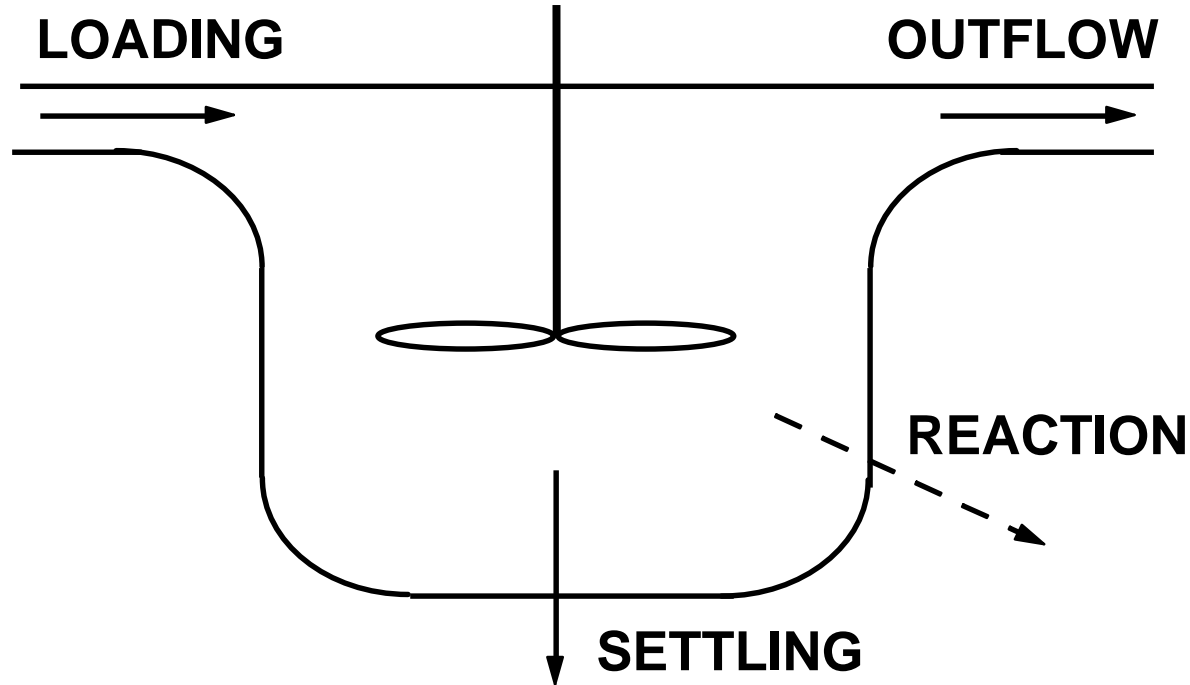
### **Mass Balance:**

*For a finite volume over a unit time period:*

$$(\text{accumulation}) = (\text{sources}) - (\text{sinks})$$



# ***MASS BALANCE FOR WELL-MIXED LAKE***



$$\text{ACCUMULATION} = \text{LOADING} - \text{OUTFLOW} - \text{REACTION} - \text{SETTLING}$$



# PARAMETERIZE ACCUMULATION

$$\text{accumulation} = \frac{\Delta M}{\Delta t}$$

Substitute  $M = Vc$

$$\text{accumulation} = \frac{\Delta Vc}{\Delta t}$$

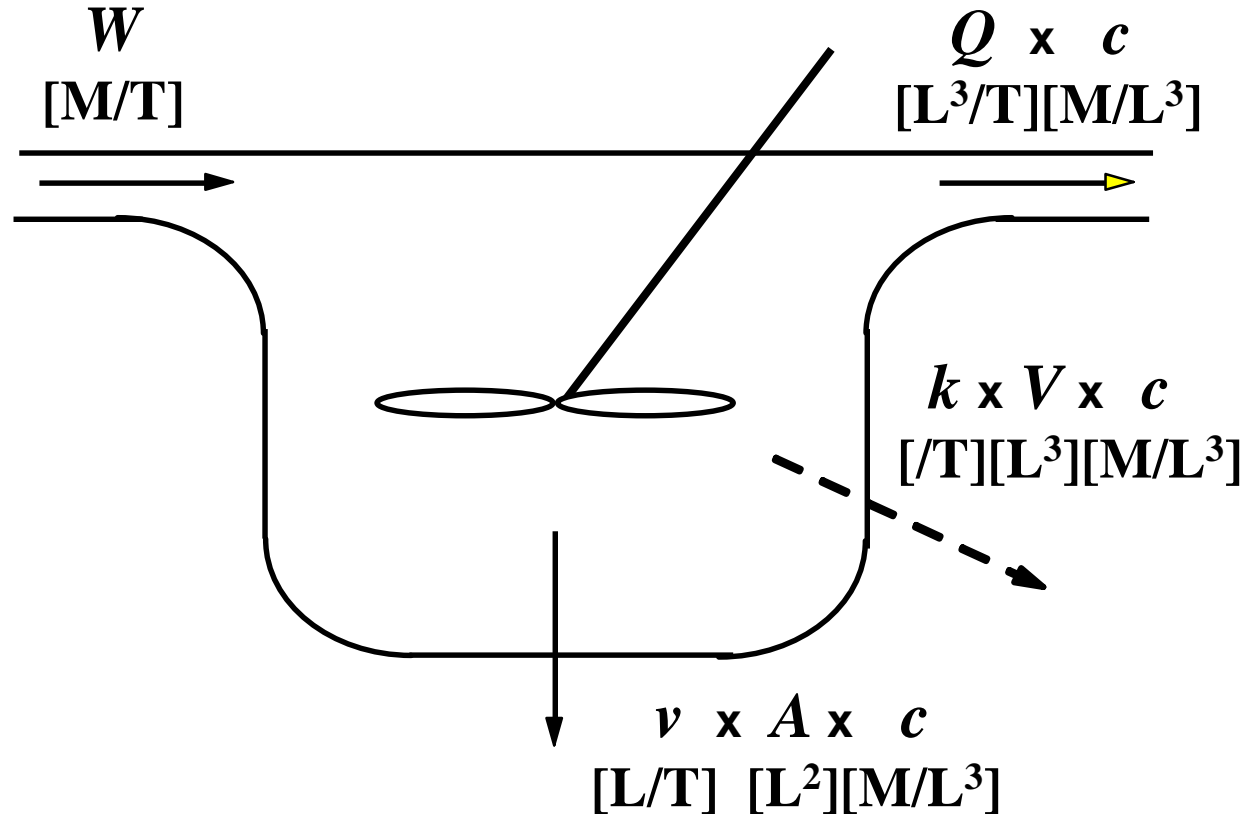
If  $V = \text{constant}$

$$\text{accumulation} = V \frac{\overset{\curvearrowright}{\Delta c}}{\Delta t}$$

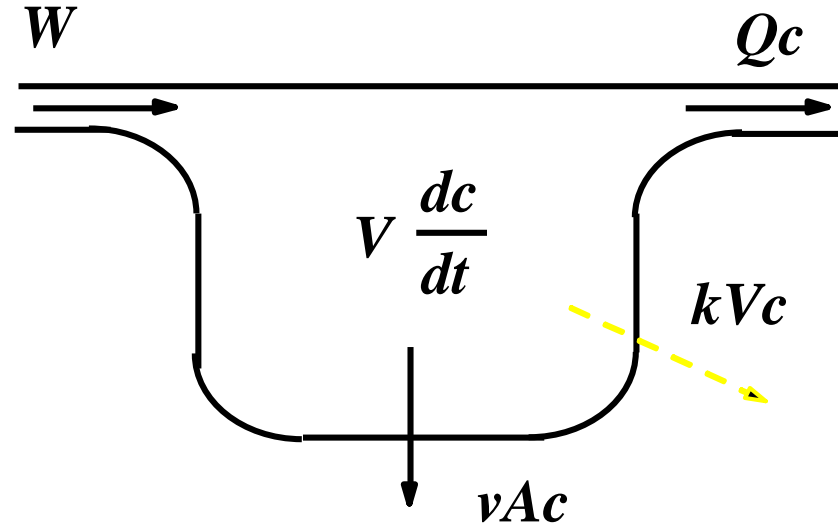
Let  $\Delta t \longrightarrow 0$

$$\text{accumulation} = V \frac{dc}{dt}$$

# PARAMETERIZE SOURCE/SINK MECHANISMS



# MASS-BALANCE EQUATION



$$V \frac{dc}{dt} = W - Qc - kVc - vAc$$

**accumulation = load – outflow – reaction – settling**

# THE FINAL MODEL

*A single, linear, first-order, nonhomogeneous ordinary differential equation*

$$V \frac{dc}{dt} = W - Qc - kVc - vAc$$

**Canonical form:**

The diagram shows the canonical form equation:  $\frac{dc}{dt} + \left( \frac{Q}{V} + k + \frac{v}{H} \right) c = \frac{W}{V}$ . Arrows point from labels to parts of the equation: 'dependent variable' points to  $c$ ; 'independent variable' points to  $t$ ; 'parameters' points to the terms  $Q/V$ ,  $k$ , and  $v/H$  inside the parentheses; and 'forcing function' points to  $W/V$ .

$$\frac{dc}{dt} + \left( \frac{Q}{V} + k + \frac{v}{H} \right) c = \frac{W}{V}$$

**dependent variable**

**independent variable**

**parameters**

**forcing function**

# SOLUTIONS

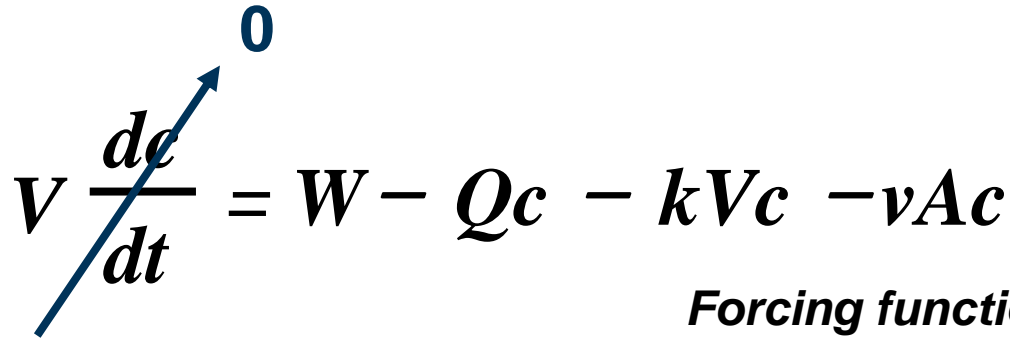
## ☀ ***Steady-state (Accumulation = 0)***

***What happens if we subject a system to constant conditions for a sufficiently long period? What will be the stable state?***

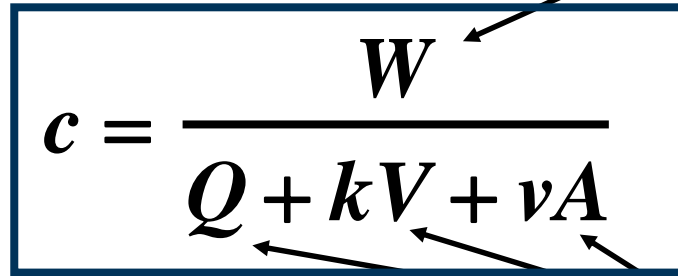
## ☀ ***Time-variable or transient solutions***

***Given an initial condition, how will the system change temporally?***

# STEADY-STATE SOLUTION


$$V \frac{dc}{dt} = W - Qc - kVc - vAc$$

*Forcing function  
(The “external”  
world)*


$$c = \frac{W}{Q + kV + vA}$$

*Purging parameters  
(physics, chemistry,  
biology)*

# WE'VE NOW DERIVED THE ASSIMILATION FACTOR!!!

$$c = \frac{W}{Q + kV + vA}$$

$$c = \frac{1}{a} W$$

$a$  = *assimilation factor*

$$= Q + kV + vA$$

*physics*

*chemistry, biology*

“transport”

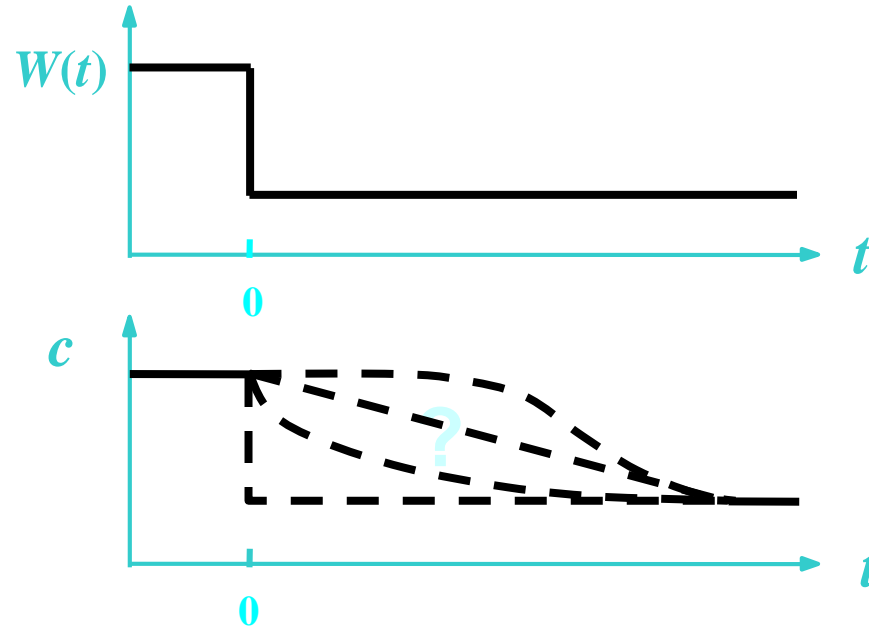
“kinetics”



# TEMPORAL ASPECTS OF POLLUTANT REDUCTION

**Questions:**

- *How long will it take for improvement to occur?*
- *What will the shape of the recovery look like?*



# ***TIME-VARIABLE SOLUTIONS ANALYTICAL VS. NUMERICAL***

$$\frac{dc}{dt} + \lambda c = \frac{W(t)}{V}$$

**analytical (closed form)**

**calculus**

**continuous**

**equation**

**numerical**

**computer**

**discrete**

**table of values**

# ***IF WE HAVE COMPUTERS: WHY ANALYTICAL SOLUTIONS???***

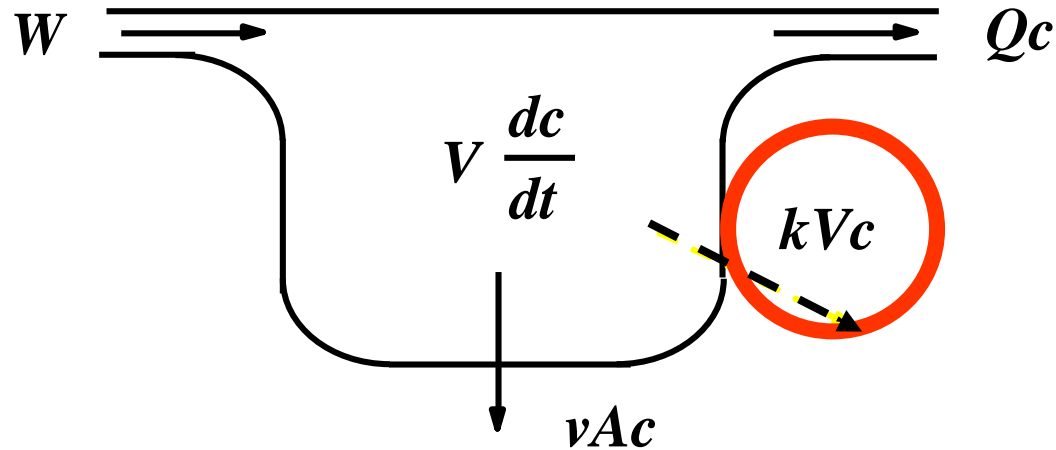
- ☀ **Great for insights and fundamental understanding**
  - ☀ For example, dimensionless numbers
- ☀ **Quick assessments**
  - ☀ “Back of the envelope” calculations
- ☀ **Useful for checking numerical computer solutions**

## ***Kinetics***

-  ***Zero & first-order reactions***

-  ***Temperature effects***

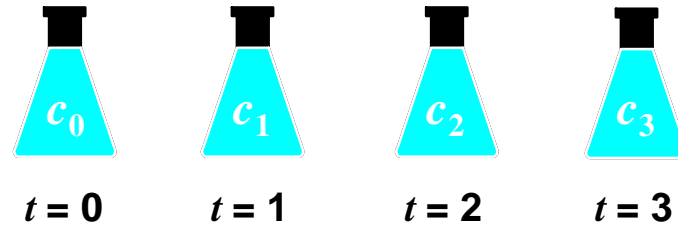
# MASS-BALANCE FOR WELL-MIXED LAKE



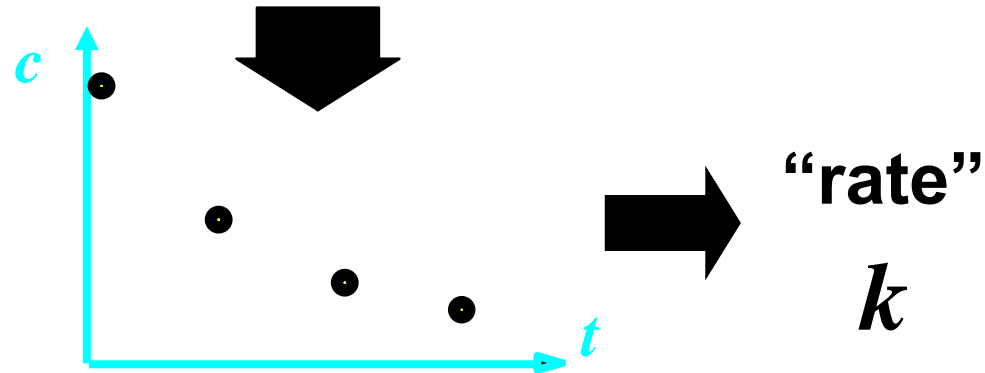
$$V \frac{dc}{dt} = W - Q_c - kVc - vAc$$

**accumulation = load – outflow – reaction – settling**

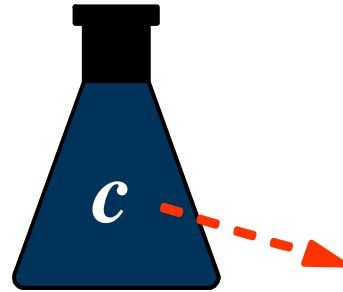
# RATE EXPERIMENT



time	0	1	2	3
concentration	$c_1$	$c_2$	$c_3$	$c_4$



# REACTION MODEL



$c$   end products

Mass balance:  $\frac{dc}{dt} = -kc^n$

$k$  is labeled "reaction 'rate'"

$n$  is labeled "reaction 'order'"



# REACTION ORDER

Zero-order reaction ( $n = 0$ )

$$\frac{dc}{dt} = -k$$

where  $k$  = zero-order reaction rate [(mg/L)/d]

-----

First-order reaction ( $n = 1$ )

$$\frac{dc}{dt} = -k c$$

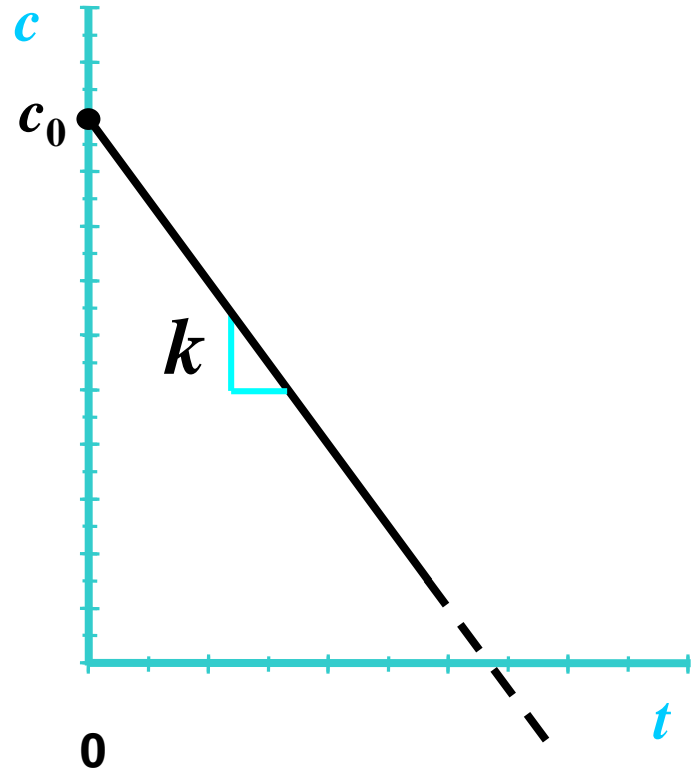
where  $k$  = first-order reaction rate [/d = per day]

# ZERO-ORDER REACTION TUM

$$\frac{dc}{dt} = -k$$

where  $c = c_0$  at  $t = 0$ ,

$$c = c_0 - kt$$



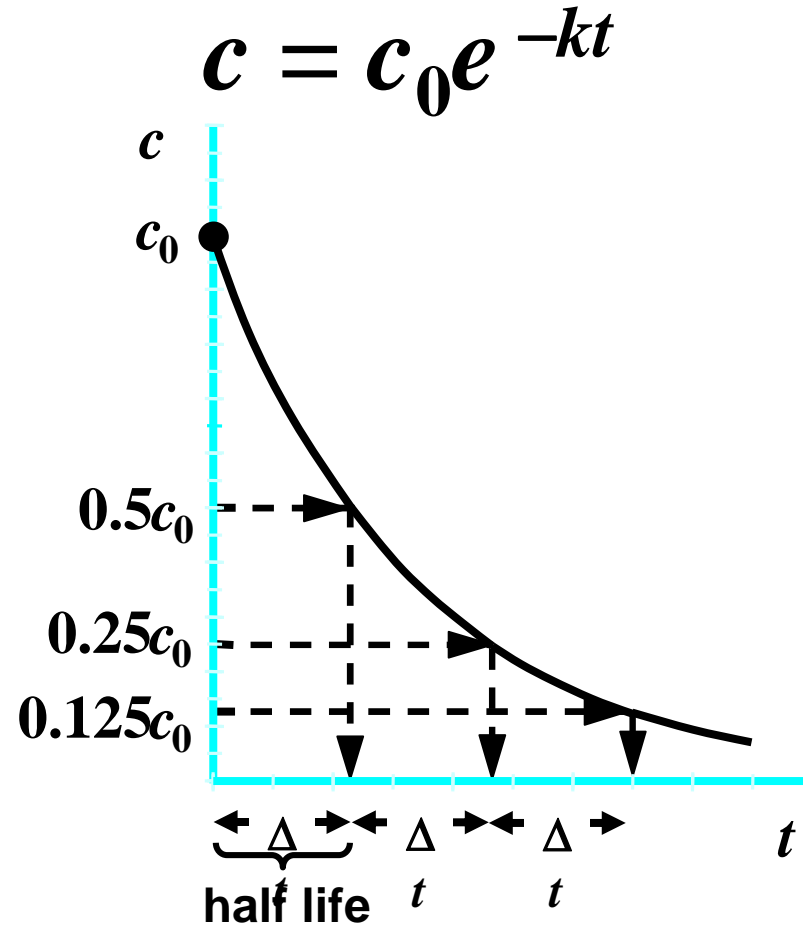
# ***FIRST-ORDER REACTION***

$$\frac{dc}{dt} = -kc$$

**where  $c = c_0$  at  $t = 0$**

$$c = c_0 e^{-kt}$$

# FIRST-ORDER REACTIONS



# THE “MEANING” OF A FIRST-ORDER RATE COEFFICIENT

If its magnitude is less than 0.5,  $k$  can be interpreted as the fraction of the pollutant that is lost per unit time.

$$0.25/\text{d} \longrightarrow 25\%/\text{d}$$

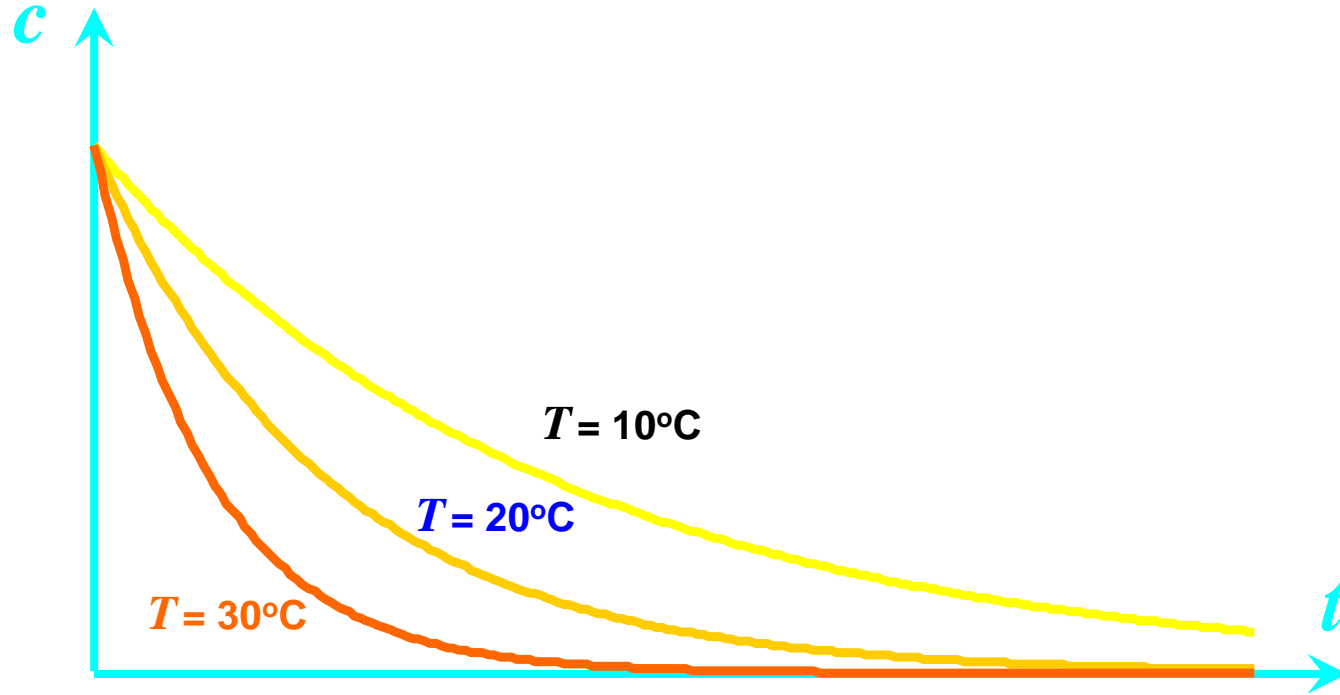
$$0.1/\text{yr} \longrightarrow 10\%/\text{yr}$$

$$2.4/\text{d} \longrightarrow 240\%/\text{d} ???$$

change units so  $< 0.5$

$$\frac{2.4}{\cancel{\text{d}}} \frac{\cancel{\text{d}}}{24 \text{ hr}} = 0.1/\text{hr} \longrightarrow 10\%/\text{hr}$$

# TEMPERATURE EFFECTS



# TEMPERATURE EFFECTS



$$\frac{k(T_2)}{k(T_1)} = \theta^{T_2 - T_1}$$

***Reference temperature***

***Chemical engineering: 25°C***

***Environmental engineering: 20°C***

$$k(T) = k(20) \theta^{T - 20}$$



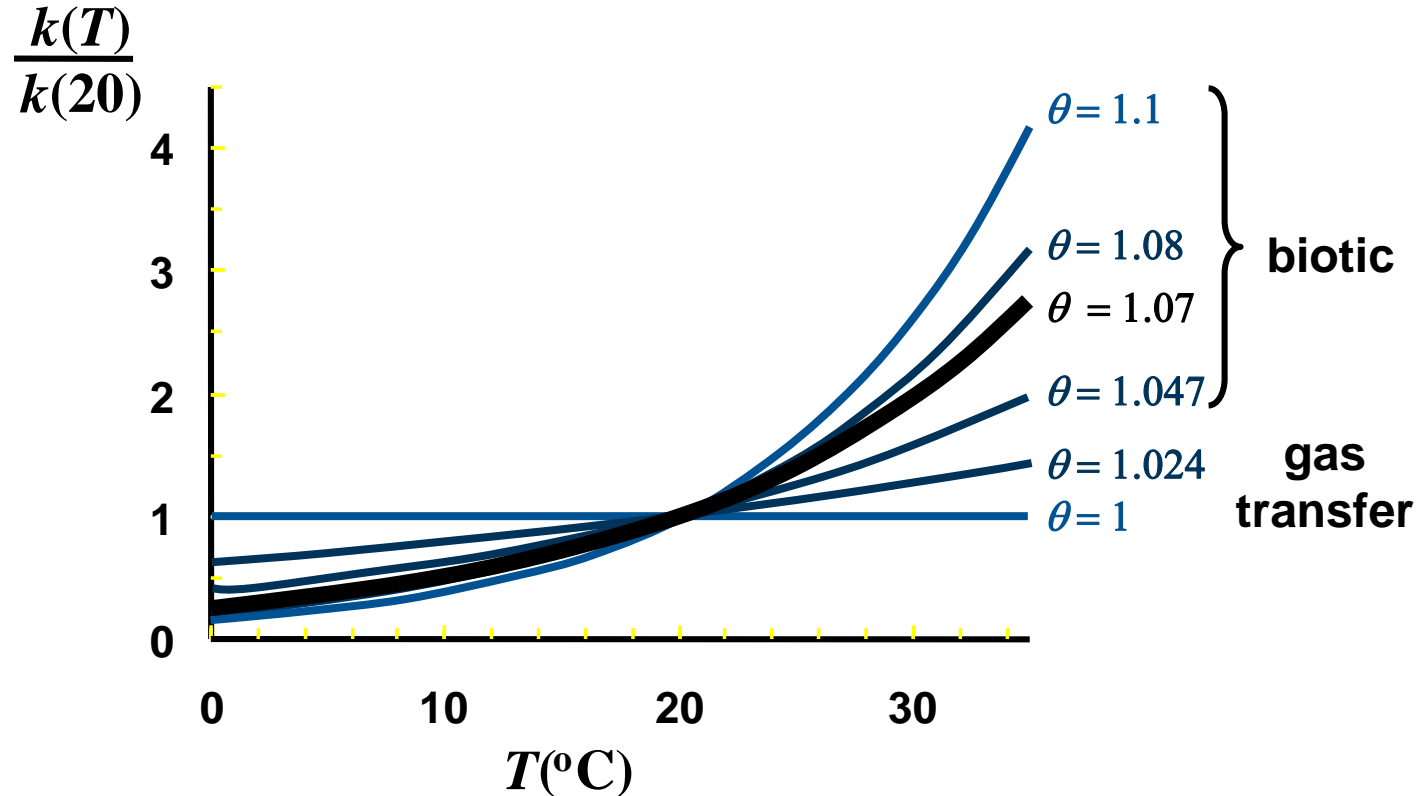
- ☀ ***Used widely by biologists***
- ☀ ***How much the rate changes per 10 °C rise in temperature***

$$Q_{10} = \frac{k(20)}{k(10)} = \theta^{10}$$



$\theta$	$Q_{10}$	Reaction
1.024	1.27	Oxygen reaeration
1.047	1.58	BOD decomposition
1.066	1.89	Phytoplankton growth
<b>1.07</b>	<b>2</b>	<b>Biological “rule of thumb”</b>
1.08	2.16	Sediment oxygen demand

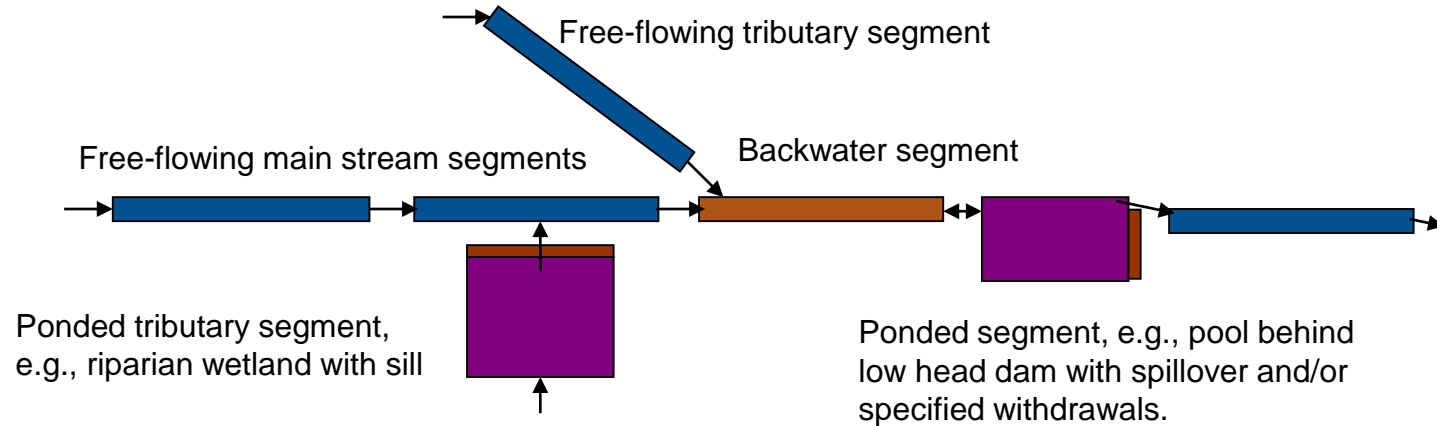
# TEMPERATURE EFFECT ON THE REACTION RATE



# ***P3. Application***

- ☀ ***0-D, 1-D, 2-D, 3-D***
- ☀ ***Water Quality Problems***
- ☀ ***WQ modelling application under climate change***

# 0-D, 1-D, 2-D, 3-D



WASP internally calculates flows, volumes, depths, widths in: free-flowing reaches (kinematic wave eq.), ponded reaches (weir overflow eq.), and backwater reaches (dynamic flow eq's.)

# Water Quality Problems

- Conventional Water Quality: DO, Eutrophication, Temperature
- Other WQ: Organic Chemicals, Nano Materials, Simple Metals, Mercury, Pathogen



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# Water quality under climate change TUM

ENVIRONMENTAL  
Science & Technology

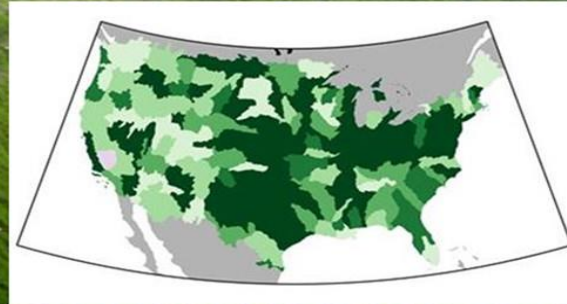
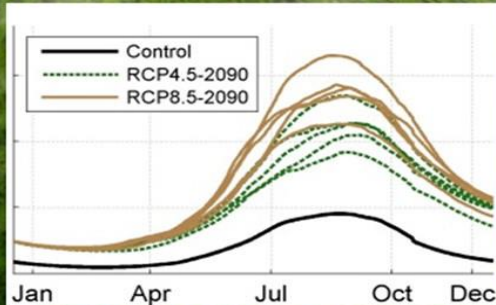


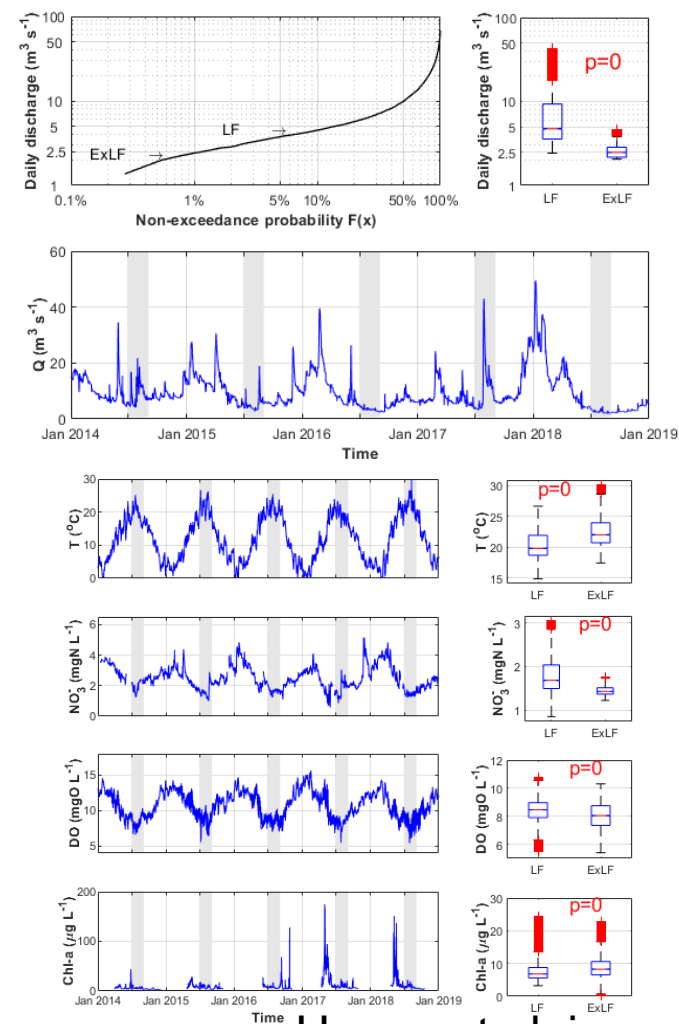
Article

[pubs.acs.org/est](https://pubs.acs.org/est)

## Climate Change Impacts on Harmful Algal Blooms in U.S. Freshwaters: A Screening-Level Assessment

Steven C. Chapra,<sup>†</sup> Brent Boehlert,<sup>\*,‡,§</sup> Charles Fant,<sup>‡</sup> Victor J. Bierman, Jr.,<sup>||</sup> Jim Henderson,<sup>⊥</sup>  
David Mills,<sup>#</sup> Diane M. L. Mas,<sup>∇</sup> Lisa Rennels,<sup>‡</sup> Lesley Jantarasami,<sup>○</sup> Jeremy Martinich,<sup>○</sup>  
Kenneth M. Strzepek,<sup>§</sup> and Hans W. Paerl<sup>∞</sup>







# *Flash flood-WQ modelling application*

**Modelling risks of infection from post-flood ponds  
exemplified by cholera in Alajo neighborhood, Accra,  
Ghana.**

**Study Project**

**Carolina Iwane Hotta**

Supervisors:

Dr. Ing. Jingshui Huang

Dr. phil. Jorge Eduardo Teixeira Leandro

Prof. Divine Kwaku Ahadzie



# Objective

## Introduction

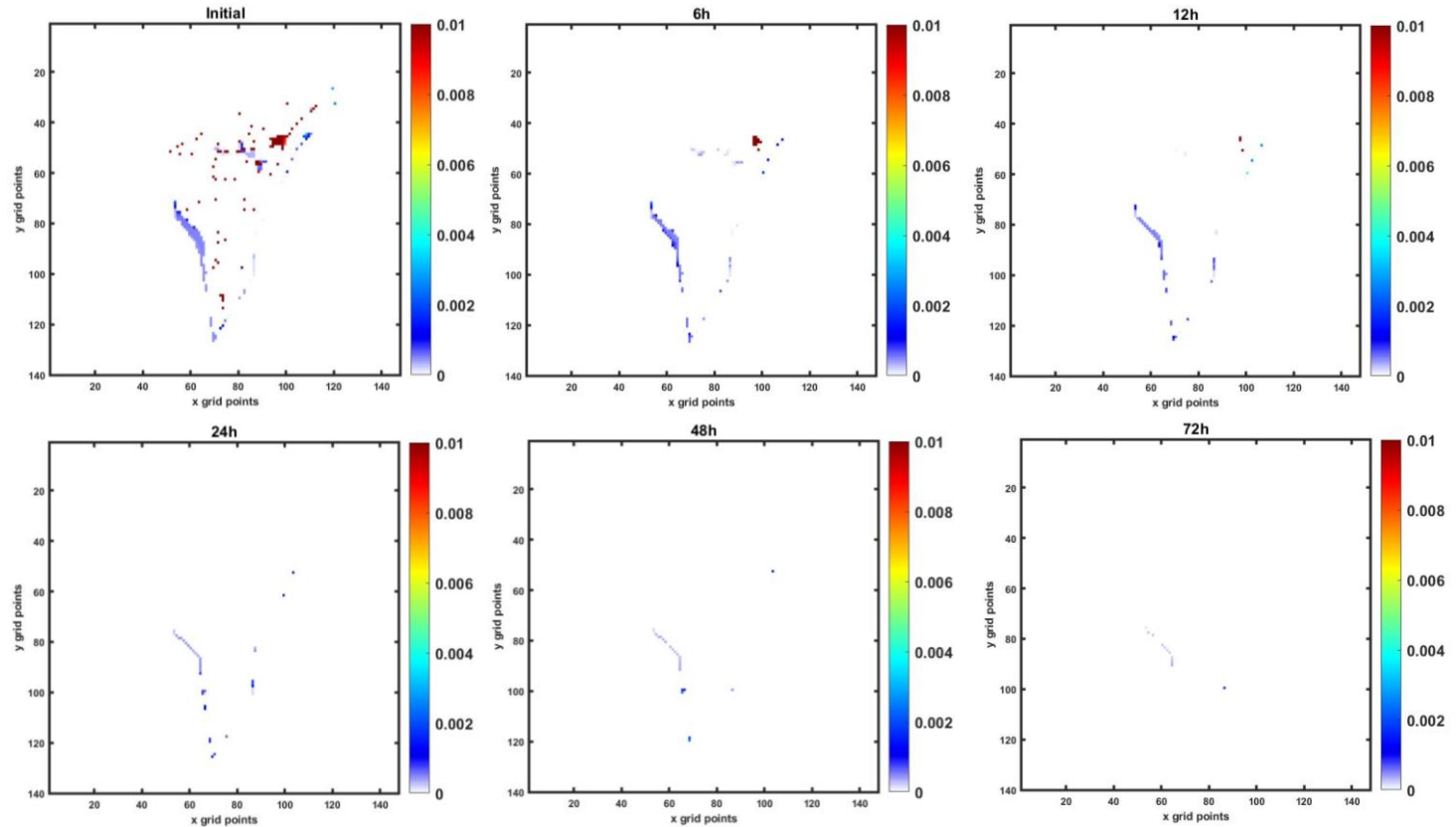


<https://www.bbc.com/pidgin/tori-47933002>

Simulate the risk for the population in Alajo, Accra, of getting infected by the bacterium *Vibrio cholerae* after possible exposure to post-flood ponds.

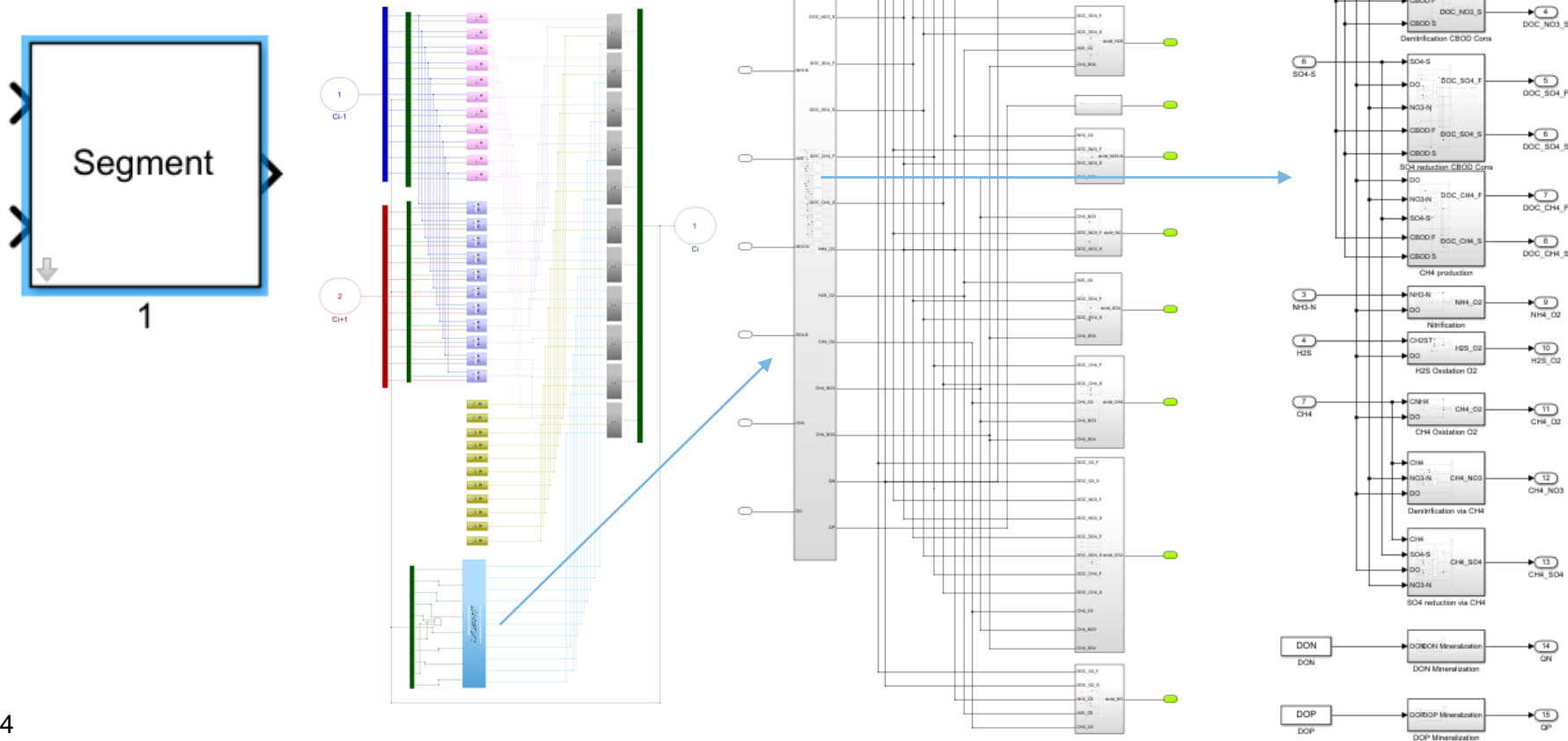
# Risk of Infection after flooding

## Results



Small children  
swimming and/or  
playing in floodwater

# SimShui

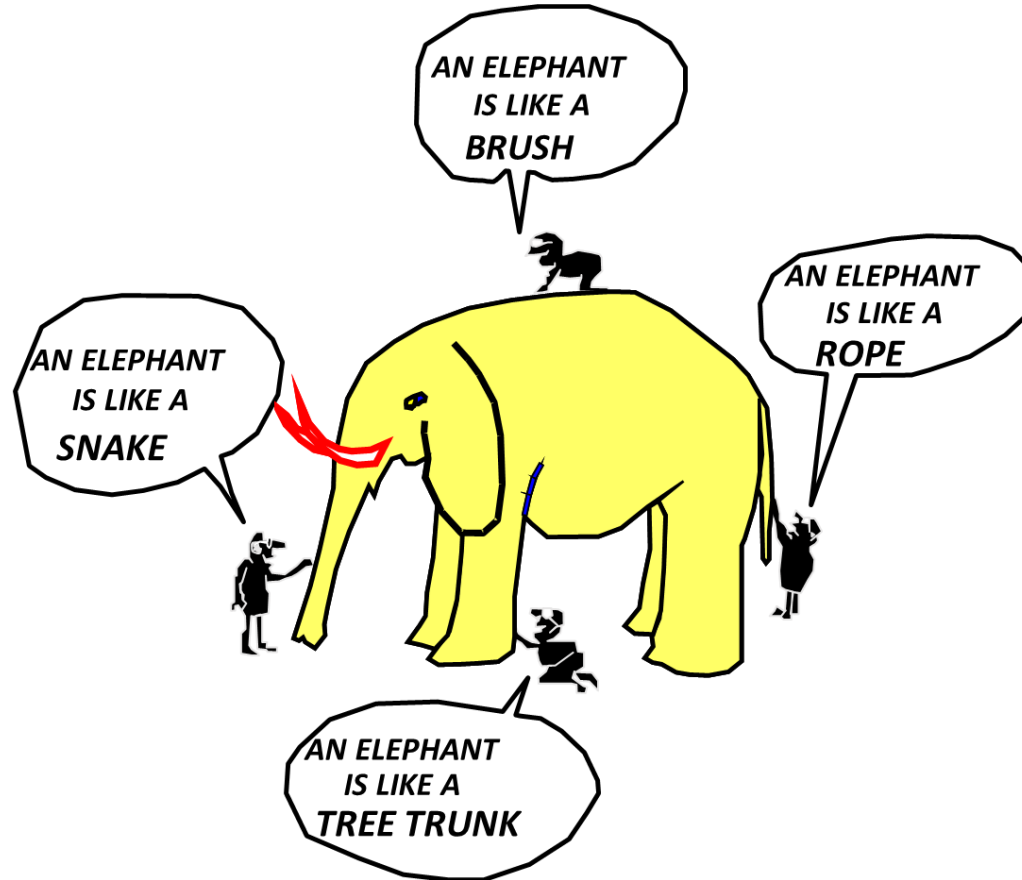




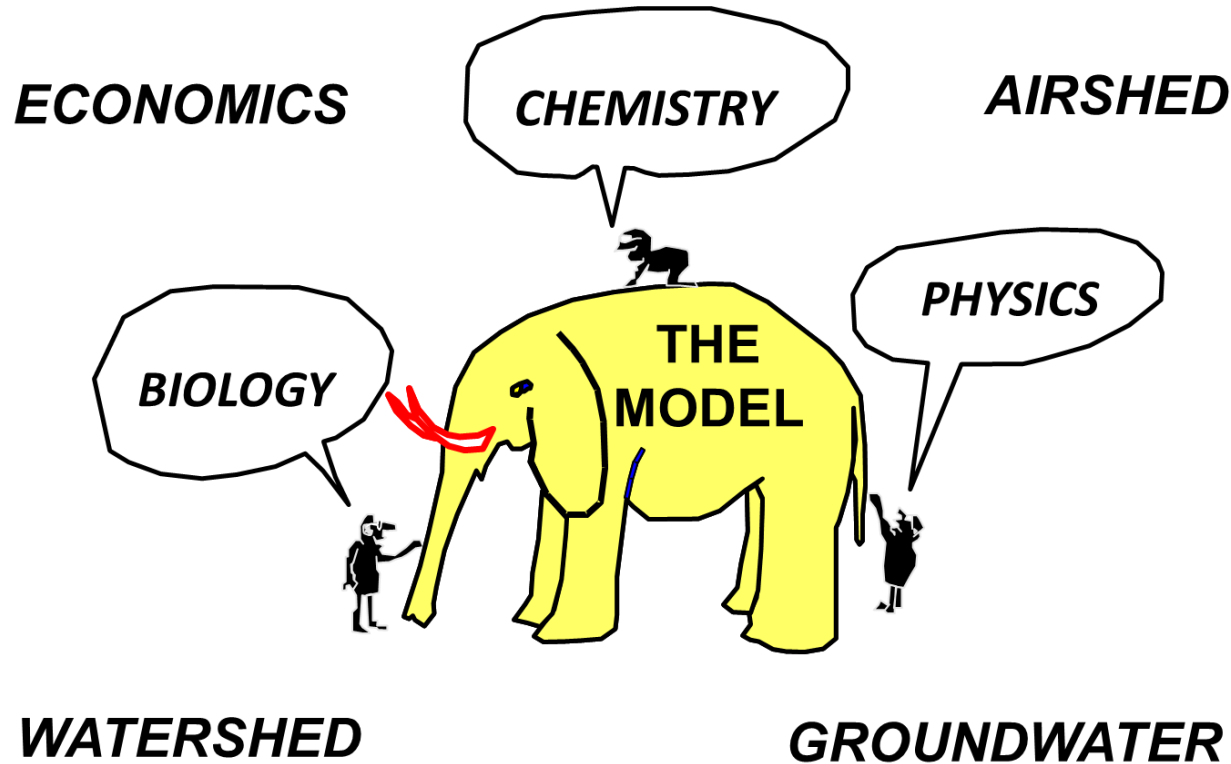
# Summary

1. Introduction to Water Quality Modeling
  - What is water quality modeling?
  - Mass balance
  - Analytical & numerical solutions
2. Kinetics
  - Zero & first-order reactions
  - Temperature effects
3. Not only water quantity, but also quality!

# MODELS GIVE YOU THE BIG PICTURE



# THE COMPUTER MODEL TIES EVERYTHING TOGETHER





Thank you very much!

TRITON





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